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Random sequential packing in square cellular structures

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Abstract. The problem of random sequential packing has been studied in statistical physics, chemical physics and biophysics. The problem has been treated mainly in the continuum or lattice space, while little attention has been paid to the packing problem in a cellular structure. As a first step to random filling in cellular structures, here we treat the problem in the square cellular structure, where squares with integer length a are inserted at random without any overlap into the cells of a square divided into square unit cells. In such packing problem, two methods A and B, which are not distinguished in the continuum space, are applied to filling squares. In A any contact among the packed squares is permitted, and forbidden in B. We calculate the packing fractions of A and B against a by computer simulation, and clearly show that the packing fraction of A is much larger than that of B when a is small. As a becomes large, the two packing fractions approach that of the continuum space, respectively, from the upper and lower side.

1. Introduction

The problem of random sequential packing (RSP) has been studied in statistical physics (Pomeau 1980, Swendsen 1981), chemical physics (Mackenzie 1962, Widom 1966, 1973, Gonzales *et al* 1974) and biophysics (Tanemura and Hasegawa 1980, Feder 1980). The RSP was executed mainly in the continuum or lattice space. Many cellular structures such as living bodies exist. However, the problem of the RSP in cellular structures or quantised spaces has had little attention. The most basic cellular structure is the square cellular structure. As a first approach to the RSP in cellular structures, here we deal with RSP problems in the square cellular structure, where squares with integer length a are inserted at random one by one without any overlap into the cells of a square divided into square unit cells. Then the sides of the inserted squares are just put on the cell boundaries of the square substrate.

In such packing problems, two methods A and B, which are not distinguished in the RSP of the continuum space, are applied to filling squares. In A any contact among the filled squares is permitted, and in B forbidden. In this paper we calculate the packing fractions (PF) of A and B against a, using computer simulation.

Many articles have been published on the problem of the RSP of orientated squares in the continuum plane. We compare the PF of A and B with previous values in the continuum plane (Akeda and Hori 1976, Feder 1980, Finegold and Donnell 1979, Jodrey and Tory 1980, Palasti 1960) and make clear the discrepancies among the RSP of A, B and the continuum plane when a is changed.

In § 2 we present examples of RSP patterns generated by A and B, and in § 3 numerical results for PF and a discussion.

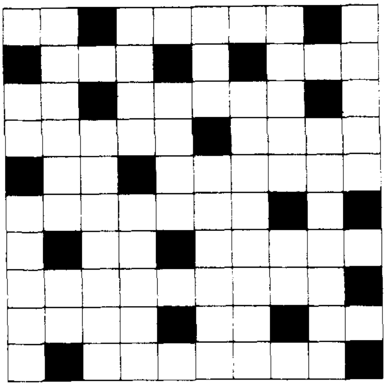

Figure 1. A random sequential packing pattern by method B for $a = 1$ and $n = 10$.

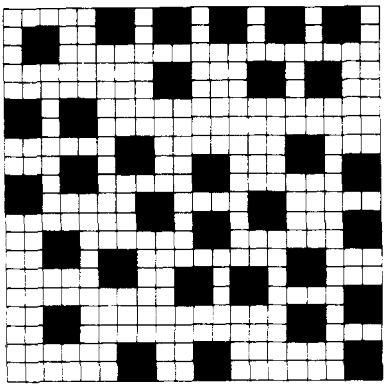

Figure 2. A random sequential packing pattern by method B for $a = 2$ and $n = 20$.

2. Random sequential packing patterns

Let the length of the square substrate be $n(\gg a)$. The computerised procedures for the RSP using methods A and B have already been presented (Nakamura 1985a, b). We applied A and B to the formation of continuum percolation patterns and irregular conducting channelling textures, respectively.

Examples of the RSP pattern of B are depicted in figures 1 and 2, for $a = 1$, $n = 10$ and $a = 2$, $n = 20$ respectively. That of A is presented in figure 3 for $a = 2$, $n = 20$. We can see in figures 2 and 3 that with equal a and n the difference between the PF of A and B is very large. The packing of figure 1 is very dilute.

3. Packing fractions and discussion

We plot the packing fractions (PF) against a/n in figures 4 and 5, for $a = 1, 2, 3, 4, 5$ and for $a = 6, 8, 10, 15, 20, 30$, respectively. Their values are the averages of fifty trials of computer simulation. In particular, we plot in figure 4, using $\times$, the averages plus or minus twice the standard deviations for $a = 1, 2$. Obviously, the PF of A for $a = 1$ is equal to unity and the standard deviation vanishes. The standard deviations for

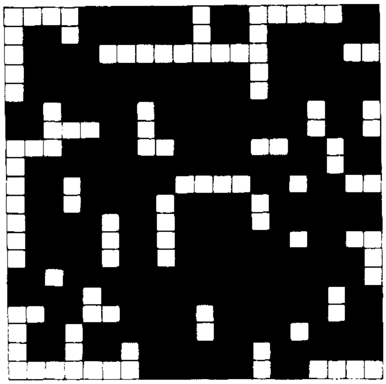

Figure 3. A random sequential packing pattern by method A for $a = 2$ and $n = 20$.

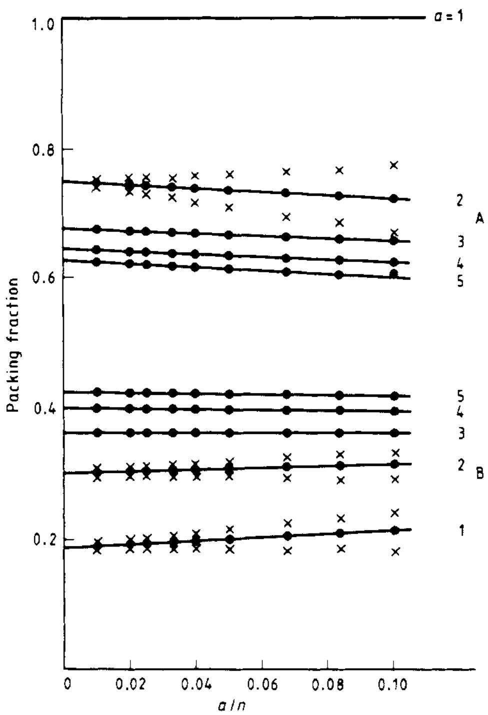

Figure 4. Packing fraction plotted against a/n for $a = 1$, 2, 3, 4 and 5. ×: ±twice standard deviation.

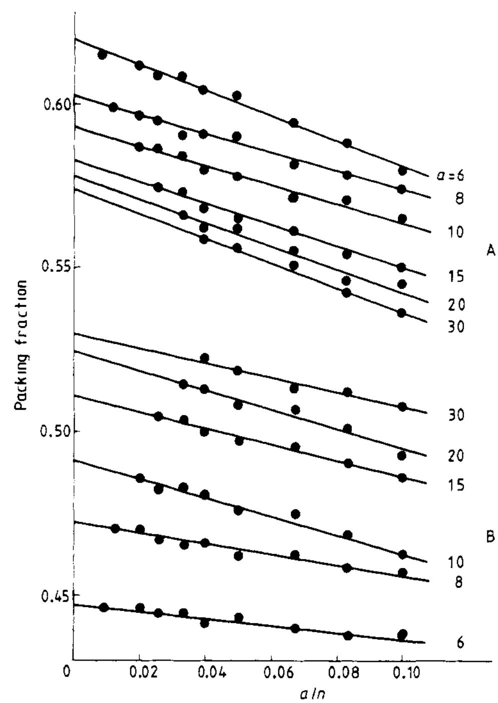

Figure 5. Packing fraction plotted against a/n for $a = 6$, 8, 10, 15, 20 and 30.

$a \geq 3$ are almost equal to those for $a \leq 2$ and they decrease with a/n. For $a \geq 3$ we do not plot the averages plus or minus twice the standard deviations because many data points are confused. The straight lines in figures 4 and 5 are determined by the linear least-squares method. The PFs of $a/n = 0$ ($n \to \infty$) can be calculated by extrapolation of the straight lines in figures 4 and 5, because the intervals of the extrapolations are much shorter than the intervals of the calculated PF and the coefficients of correlation are close to ± 1 (good fitting to a straight line) except for a few coefficients as shown in table 1. The PF calculated by the extrapolations are listed in table 1 with their standard deviations. The standard deviations are computed by the linear least-squares

Table 1. Packing fraction, standard deviation and correlation coefficient of lines determined by linear least-squares method.

	a	1	2	3	4	5	6	8	10	15	20	30
Packing fraction	A	1.000	0.749	0.681	0.646	0.628	0.620	0.603	0.593	0.583	0.578	0.574
	B	0.187	0.302	0.364	0.404	0.429	0.447	0.472	0.491	0.511	0.525	0.530
Standard deviation	A	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	B	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
Correlation coefficient	A	—	−0.969	−0.985	−0.932	−0.988	−0.996	−0.975	−0.993	−0.996	−0.980	−0.994
	B	0.998	0.981	0.459	−0.902	−0.958	−0.950	−0.991	−0.986	−0.996	−0.981	−0.986

method and extrapolation and their coefficients of correlation are close to ±1. The extrapolation method applied here was presented by Blaisdell and Solomon (1970).

We plot the PF ($n \to \infty$) of table 1 against $1/a$ in figure 6. The detail for $1/a \leqslant 0.1$ of figure 6 is presented in figure 7. The PF of $1/a \to 0$ are calculated by the linear least-squares and extrapolation method in figure 7. The two PF p_a and p_b, the standard deviations σ_a and σ_b and the correlation coefficients γ_a and γ_b of the straight lines are

$$p_a = 0.564 \tag{1}$$

$$\sigma_a = 0.001 \tag{2}$$

$$\gamma_a = 0.998 \qquad \text{for A} \tag{3}$$

and

$$p_b = 0.552 \tag{4}$$

$$\sigma_b = 0.002 \tag{5}$$

$$\gamma_b = -0.993 \qquad \text{for B.} \tag{6}$$

We compare in table 2 p_a and p_b with previous values in the continuum plane. The two values p_a and p_b agree well with the previous ones, and they are, respectively, greater or less than the previous values.

In this simulation p_a approaches p_b very well, but no complete agreement occurs. When we denote the true PF in the continuum plane by p_c, the explicit relation among

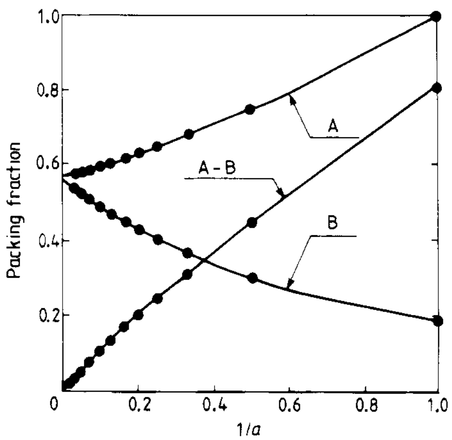

Figure 6. Packing fraction plotted against $1/a$.

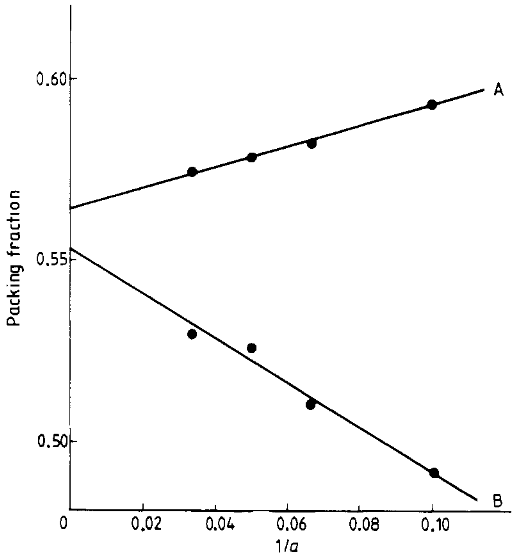

Figure 7. Detail of figure 6 when $1/a$ is close to zero.

Table 2. Comparison with previous work.

0.564	This work A
0.552	This work B
0.562	Feder (1980)
0.562 9	Akeda and Hori (1976)
0.553 8	Finegold and Donnell (1979)
0.562 10	Jodrey and Tory (1980)
0.558 9	Palasti (1960)

p_a, p_b and p_c is left open for future work by more accurate simulation or a mathematical method.

When $a = 1$, the PF $p_{a1}(n \to \infty)$ for A is obviously 1.0 and the PF $p_{b1}(n \to \infty)$ for B is only 0.187 ± 0.002, where ± 0.002 is twice the standard deviation. The difference between the two values is

$$p_{a1} - p_{b1} = 0.813 \pm 0.002 \tag{7}$$

and the greatest. When $a = 2$, the difference of the PF is equal to 0.447 ± 0.004, and still large. As a increases, the difference becomes smaller. However, the speed of the approach is slow. In fact even when $a = 10$, the difference between the PF is equal to 0.102 ± 0.004.

When a is small, the PF in the cellular structure are very different from the PF in the continuum plane. Then the PF by A and B are, respectively, much larger and smaller than the PF in the continuum plane.

We applied the RSP probem in the square cellular structure to the continuum percolation problem in two dimensions (Nakamura 1985a, b). The problem is applicable to the absorption and territory in cellular structures.

In this paper we have obtained the PF only by computer simulation, but it is much better to obtain the exact PF by mathematical means. This problem remains for future work.

4. Conclusions

In this paper we studied random sequential packing (RSP) problems in the square cellular structure. We inserted, at random, one by one squares with integer length a into the cells of a square substrate divided into unit square cells. To insert the squares, we applied two methods A and B, which were not distinguished in the RSP of the continuum space. In A any contact among the packed square was permitted, and forbidden in B. We computed by computer simulation the packing fractions (PF) of A and B against a. The results were plotted in figures 6 and 7.

We calculate the PF of A and B at the limit of $a \to \infty$ by an extrapolation method. We compare in table 2 the two PF with previous values for the continuum plane. They agreed well with the previous results and the PF of A and B at $a \to \infty$ are, respectively, larger and smaller than the previous values for the continuum plane.

As a is small, the discrepancies among the PF of A, B and the continuum plane become substantial. The PF of A and B become, respectively, much larger and smaller than the PF of the continuum plane.

The mathematical treatment of the RSP in the cellular structure remains for future work.

Acknowledgments

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